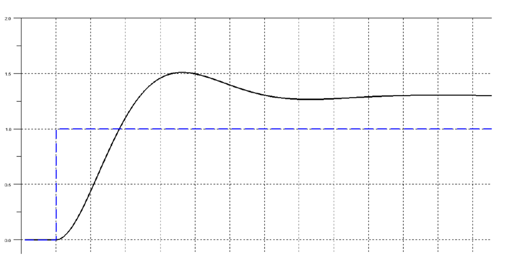
Ex 1, figure 2



1.- 4. : Damped 2nd order system without ideal delay.

$K \simeq 1.3$, $\xi \simeq 0.5$ and $\omega_n \simeq 0.5$

in $H(s) = \dfrac{K}{1 + \frac{2\xi}{\omega_n} s + \frac{1}{\omega_n^2} s^2}$

$\Delta = 1$ is an appropriate sampling period.

5. The discrete-time transfer function of a 2nd order system (see p. 12 of the manuscript):

$$H(z) = \frac{b_1 z + b_0}{z^2 + a_1 z + a_0}$$

with (since $\xi < 1$):

if $\xi < 1$ noting $\alpha = e^{-\xi\omega_n\Delta}$, $\omega_p = \omega_n\sqrt{1-\xi^2}$
$a_0 = \alpha^2$ $a_1 = -2\alpha\cos(\omega_p\Delta)$ $b_0 = \alpha^2 + \alpha\left[\xi\frac{\omega_n}{\omega_p}\sin(\omega_p\Delta) - \cos(\omega_p\Delta)\right]$ $b_1 = 1 - \alpha\left[\xi\frac{\omega_n}{\omega_p}\sin(\omega_p\Delta) + \cos(\omega_p\Delta)\right]$

6. Recursive equation ?

Two solutions :

a. Use the general form given in the manuscript (see p. 4 of the manuscript):

$$y(k+n) + a_{n-1}y(k+n-1) + \ldots + a_0 y(k) = b_n u(k+n) + b_{n-1}u(k+n-1) + \ldots + b_0 u(k)$$

and specialize it with $n=2$ (order of the system).

b. Similarly as in continuous time, we can get the recursive equation from the transfer function using inverse $\mathcal{Z}$ transform (instead of inverse Laplace transform in continous time).

Reminders :

Continuous-time

temporal domain	frequential domain
$f(t)$ $\underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}}$	$F(s)$
$f(t) + g(t)$	$F(s) + G(s)$
$\lambda f(t)$	$\lambda F(s)$
$f^{(n)}(t)$	$s^n F(s)$

n-th derivative of f

Discrete-time

temp. dom.	freq. dom.
$f(k)$ $\underset{\mathcal{Z}^{-1}}{\overset{\mathcal{Z}}{\rightleftarrows}}$	$F(z)$
$f(k) + g(k)$	$F(z) + G(z)$
$\lambda f(k)$	$\lambda F(z)$
$f(k+l)$	$z^l F(z)$

We have

$$H(z) = \frac{Y(z)}{U(z)} = \frac{b_1 z + b_0}{z^2 + a_1 z + a_0}$$

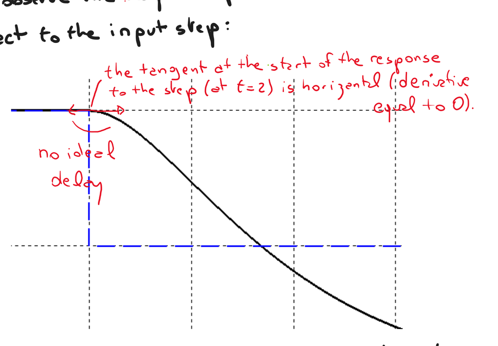
$$\Leftrightarrow z^2 Y(z) + a_1 z Y(z) + a_0 Y(z) = b_1 z U(z) + b_0 U(z)$$

$\mathcal{Z}^{-1}$

$$y(k+2) + a_1 y(k+1) + a_0 y(k) = b_1 u(k+1) + b_0 u(k)$$

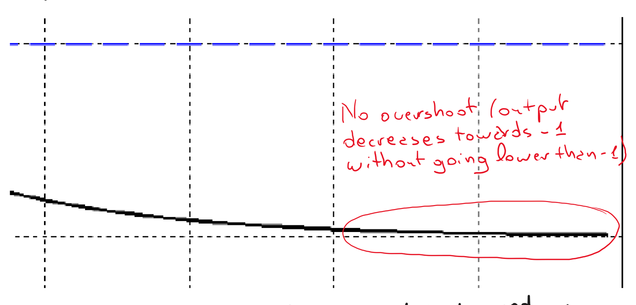
Ex 1, figure 4

1. We observe the output response when it starts to react to the input step:



From the observations above, we can say the order of the system is greater or equal to 2 (here approximated to 2) and there is no ideal delay.

In addition, one can observe the continuation of the output response:



There is no overshoot, which means that the 2nd order is aperiodic (i.e. damping ratio ξ is greater or equal to 1).

2. A continuous-time transfer function for such a system is:

$$H(s) = K\frac{1}{1 + 2\xi\frac{s}{\omega_n} + (\frac{s}{\omega_n})^2}$$

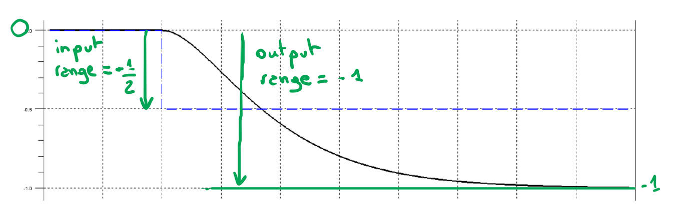
Since the system is aperiodic, we know that it can be written:

$$H(s) = \frac{K}{(1+T_1 s)(1+T_2 s)} \quad \text{(p. 11-12)}$$

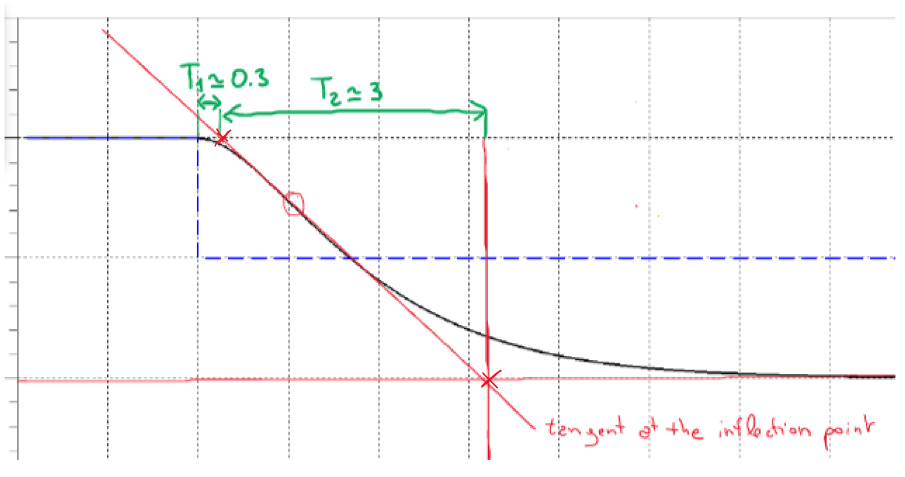
with

$$\xi = \frac{1}{2}\frac{T_1+T_2}{\sqrt{T_1T_2}} \text{ and } \omega_n = \frac{1}{\sqrt{T_1T_2}}$$

3. To estimate the parameters K, ξ and ω_n, we make measurements as follows:



Gain $K = \frac{-1}{-1/2} = 2$



Damping ratio $\xi = \frac{1}{2}\frac{T_1+T_2}{\sqrt{T_1T_2}} \simeq 1.8$

Natural frequency $\omega_n = \frac{1}{\sqrt{T_1T_2}} \simeq 1.1$

4. The sampling period Δ has to be chosen to satisfy :

$$0.25 < \Delta\omega_n < 1.25 \quad \text{(rule for 2nd order system)}$$

Here, $\Delta = 0.5$ can be appropriate.

5. The discrete-time transfer function of a 2nd order system is (see p. 12 of the manuscript):

$$H(z) = \frac{b_1 z + b_0}{z^2 + a_1 z + a_0}$$

with (since $\xi \geq 1$):

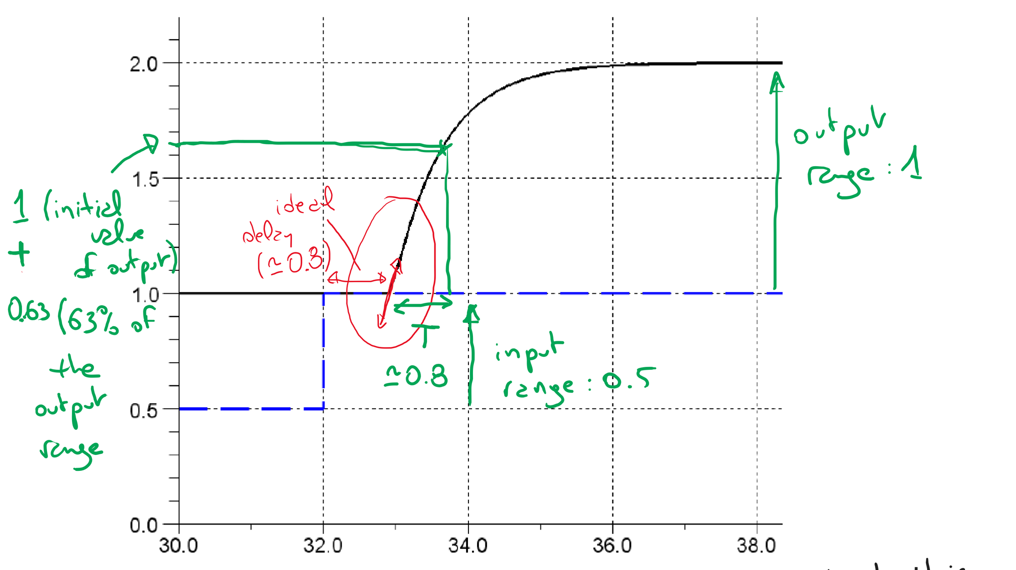
$a_0 = z_1 z_2$ $\qquad$ $b_0 = K(z_1 z_2 - \frac{T_1 z_2 - T_2 z_1}{T_1 - T_2})$

$a_1 = -(z_1 + z_2)$ $\qquad$ $b_1 = K(\frac{1 - T_1 z_1 - T_2 z_2}{T_1 - T_2})$

$$\text{noting } z_1 = e^{-\Delta/T_1},\ z_2 = e^{-\Delta/T_2}$$

6. See the correction for figure 2.

Exercice 2



1) . non-horizontal tangent : 1st order system (no overshoot, this confirms that it can be a 1st order)

. initial delay

2). Continuous-time transfer function:

$$H(s) = \frac{K}{1+Ts} e^{-\tau s}$$

Gain

delay

Time constant

3). $K = \frac{1}{0.5} = 2$; $T \simeq 0.8$, $\tau \simeq 0.8$

4) For 1st order, the rule for the choice of the sampling period Δ is :

$$0.25T < \Delta < 1.25T$$

Here, we have to choose Δ so that:

$$0.2 < \Delta < 1$$

$\Delta = 0.5$ seems to be a good choice here.