Control theory: state-space approach for linear systems

State observer

2 janvier 2022

Introduction

Until then, we have assumed that all the state-variables could be measured (to be used to implement the state-feedback control).

Introduction

Until then, we have assumed that all the state-variables could be measured (to be used to implement the state-feedback control).

Most of the time, either due to "physical" constraints or because it is costly, we do not add dedicated sensors to measure state variables.

Introduction

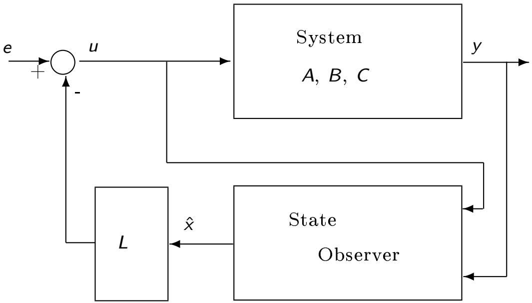
Until then, we have assumed that all the state-variables could be measured (to be used to implement the state-feedback control).

Most of the time, either due to "physical" constraints or because it is costly, we do not add dedicated sensors to measure state variables.
A state observer is a system that provides an estimate $\hat{x}$ of the internal state x of a given real system, from measurements of the input and output of the real system. It is typically computer-implemented, and provides the basis of many practical applications.

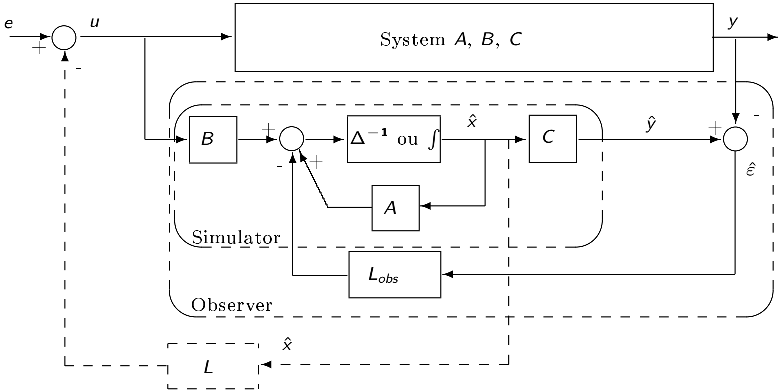
Introduction (continued)

For state-feedback control : estimated state $\hat{x}$ is weighted by regulation matrix L to get the control input

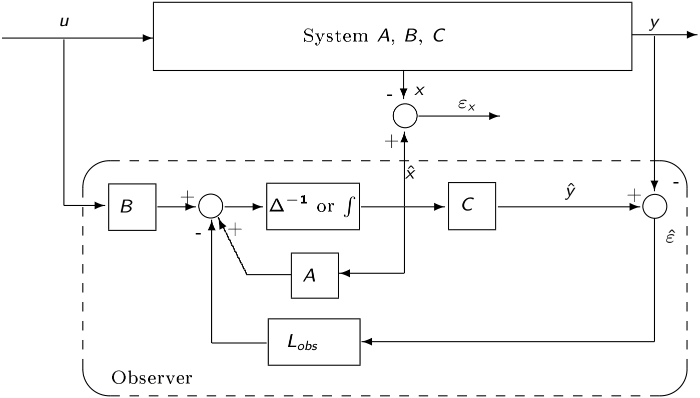
$$u = e - L\hat{x}.$$



State observer principle



Synthesis of matrix L_{obs}



To find L_{obs}, we focus on system with "input" $u(t)$ and "output" $\varepsilon_x(t) = \hat{x}(t) - x(t)$.

Synthesis of matrix L_{obs} (continued)

We have

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) \\ \frac{d}{dt}(\hat{x}(t)) & = & A\hat{x}(t) + Bu(t) - L_{obs}(C\hat{x}(t) - Cx(t)) \end{array} \right. .$$

Synthesis of matrix L_{obs} (continued)

We have

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) \\ \frac{d}{dt}(\hat{x}(t)) & = & A\hat{x}(t) + Bu(t) - L_{obs}(C\hat{x}(t) - Cx(t)) \end{array} \right. .$$

hence

$$\begin{array}{rcl} \dot{\varepsilon}_x(t) & = & \frac{d}{dt}(\hat{x}(t) - x(t)) \\ & = & A\hat{x}(t) + Bu(t) - L_{obs}(C\hat{x}(t) - Cx(t)) - Ax(t) - Bu(t) \\ & = & A(\hat{x}(t) - x(t)) - L_{obs}C(\hat{x}(t) - x(t)) \end{array} ,$$

Synthesis of matrix L_{obs} (continued)

We have

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & Ax(t)+Bu(t) \\ \frac{d}{dt}(\hat{x}(t)) & = & A\hat{x}(t)+Bu(t)-L_{obs}(C\hat{x}(t)-Cx(t)) \end{array}\right. .$$

hence

$$\begin{array}{rcl} \dot{\varepsilon}_x(t) & = & \frac{d}{dt}(\hat{x}(t)-x(t)) \\ & = & A\hat{x}(t)+Bu(t)-L_{obs}(C\hat{x}(t)-Cx(t))-Ax(t)-Bu(t) \\ & = & A(\hat{x}(t)-x(t))-L_{obs}C(\hat{x}(t)-x(t)) \end{array} ,$$

or

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & Ax(t)+Bu(t) \\ \dot{\varepsilon}_x(t) & = & A\varepsilon_x(t)-L_{obs}C\varepsilon_x(t) \end{array}\right. .$$

Synthesis of matrix L_{obs} (continued)

State-rep. of system with "input" $u(t)$ and "output" $\varepsilon_x(t)$

$$\left\{\begin{array}{rcl} \begin{pmatrix} \dot{x}(t) \\ \dot{\varepsilon}_x(t) \end{pmatrix} & = & \begin{pmatrix} A & 0 \\ 0 & A - L_{obs} C \end{pmatrix} \begin{pmatrix} x(t) \\ \varepsilon_x(t) \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u(t) \\ \varepsilon_x(t) & = & \begin{pmatrix} 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} x(t) \\ \varepsilon_x(t) \end{pmatrix} \end{array}\right.$$

Synthesis of matrix L_{obs} (continued)

State-rep. of system with "input" $u(t)$ and "output" $\varepsilon_x(t)$

$$\begin{cases} \begin{pmatrix} \dot{x}(t) \\ \dot{\varepsilon}_x(t) \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & A - L_{obs}C \end{pmatrix} \begin{pmatrix} x(t) \\ \varepsilon_x(t) \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u(t) \\ \varepsilon_x(t) = \begin{pmatrix} 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} x(t) \\ \varepsilon_x(t) \end{pmatrix} \end{cases}$$

- $u(t)$ has no influence on subvector $\varepsilon_x(t)$ (described by $\dot{\varepsilon}_x(t) = (A - L_{obs}C)\varepsilon_x(t)$).

Synthesis of matrix L_{obs} (continued)

State-rep. of system with "input" $u(t)$ and "output" $\varepsilon_x(t)$

$$\begin{cases} \begin{pmatrix} \dot{x}(t) \\ \dot{\varepsilon}_x(t) \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & A - L_{obs}C \end{pmatrix} \begin{pmatrix} x(t) \\ \varepsilon_x(t) \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u(t) \\ \varepsilon_x(t) = \begin{pmatrix} 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} x(t) \\ \varepsilon_x(t) \end{pmatrix} \end{cases}$$

- $u(t)$ has no influence on subvector $\varepsilon_x(t)$ (described by $\dot{\varepsilon}_x(t) = (A - L_{obs}C)\varepsilon_x(t)$).
- *Observation error* $\varepsilon_x(t)$ depends on poles of $(A - L_{obs}C)$.

Synthesis of matrix L_{obs} (continued)

State-rep. of system with "input" $u(t)$ and "output" $\varepsilon_x(t)$

$$\left\{\begin{array}{rcl} \begin{pmatrix} \dot{x}(t) \\ \dot{\varepsilon}_x(t) \end{pmatrix} & = & \begin{pmatrix} A & 0 \\ 0 & A - L_{obs}C \end{pmatrix} \begin{pmatrix} x(t) \\ \varepsilon_x(t) \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u(t) \\ \varepsilon_x(t) & = & \begin{pmatrix} 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} x(t) \\ \varepsilon_x(t) \end{pmatrix} \end{array}\right.$$

- $u(t)$ has no influence on subvector $\varepsilon_x(t)$ (described by $\dot{\varepsilon}_x(t) = (A - L_{obs}C)\varepsilon_x(t)$).
- *Observation error* $\varepsilon_x(t)$ depends on poles of $(A - L_{obs}C)$.
- This leads to solve

$$|s\mathbf{I} - A + L_{obs}C| = P_{obs}(s) \Leftrightarrow |s\mathbf{I} - A^\top + C^\top L_{obs}^\top| = P_{obs}(s)$$

in which $P_{obs}(s)$ is chosen : **pole placement equation**.