MATHEMATICAL MODELS IN RELIABILITY

1. Quantitative reliability measures

State variable: $X(t) = \begin{cases} 1 & \text{if the system is on at } t \\ 0 & \text{if the system is out at } t \end{cases}$

Time to failure (life time, operating time): *time elapsing from when the system is put on operation until it fails its first time*

- Random variable: discrete or continuous
  - A submarine mission time
  - Number of washing cycles of a washing machine
  - Number of kilometers for a tyre
  - Number of flights for a landing system of a plane
  - Operating time for electronical devices
  - Number of shocks for a watch

If no particular mention, the time to failure will be considered as continuous

1. Quantitative reliability measures

Expressed by	$F(t)$	$f(t)$	$R(t)$	$z(t)$
$F(t) =$	–			
$f(t) =$		–		
$R(t) =$			–	
$z(t) =$				–

1. Quantitative reliability measures

Drawn the failure rate function over the life cycle

1. Quantitative reliability measures

- Is the function $1/(0.2t + 1)^2$ can be a reliability function?
- From its definition, find the different expressions of the **MTTF**
- What can represent the **Median Life** of a system?
- What is a **quantile** of the life distribution?
- What is the **Mode** of a distribution?
- How do you define the **Mean Residual Life** of a system?
  - Define the conditional survivor/reliability function
  - Write the MRL as a function of the reliability function

A park of n identical systems is inspected at time t, given that all of the systems are new at $t = 0$.

What are the parameters for the evaluation of the probability that we can observe $k < n$ failed systems ?

What are the expected number of failed systems and the associated variance?

Let assume a non-repairable system which is new at $t = 0$. At each of the failures, the system is replaced by a new identical one. Let consider a negligible replacement time.

The objective is then to estimated the number of replacements in a time interval.

Let λ be the *intensity* of the counting stochastic process $\{N(t), t \geq 0\}$ or the *frequency* of the failure event.

1. Find the probability of the number of replacements in $(0, t)$ is n
2. Find the probability of the number of replacements in $(t, t + s)$ is n given $N(t) = 0$ for any t
3. Give the expected number of replacements in $(0, t)$ and the associated variance

Definition:

Let consider an event A. Let assume that:

1. The event A may occur at any time in the interval, and the probability of A occuring in the interval $(t, t + \Delta t)$ is independent of t and may be written as $\lambda \cdot \Delta t + o(\Delta t)$ where $\lambda > 0$
2. The probability of more than one event in $(t, t + \Delta t)$ is $o(\Delta t)$.
3. Let $(t_{11}, t_{12}), (t_{21}, t_{22})$, ... be any sequence of disjoint intervals. Then the events « A occurs in (t_{j1}, t_{j2}) », $j = 1,2,$ ... are independent.

Then the stochastic process $\{N(t), t \geq 0\}$ is an Homogeneous Poisson Process (HPP) with rate λ

Discuss on the probability that $N(t) = 0$ for any given t

Let assume now that the failure rate function of a life time T is non linear, $z(t) = at^b$.

Write the reliability function of T

Discuss the model when $b = 0$

Definition

The Weibull failure model is defined when the failure time T is Weibull-distributed, i.e.

$$F_T(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}} \; for \; t > 0$$

Write the reliability function, the pdf, the failure rate function, the MTTF and the $MRL(t)$

Discuss the monotonicity and plot the failure rate when β varies

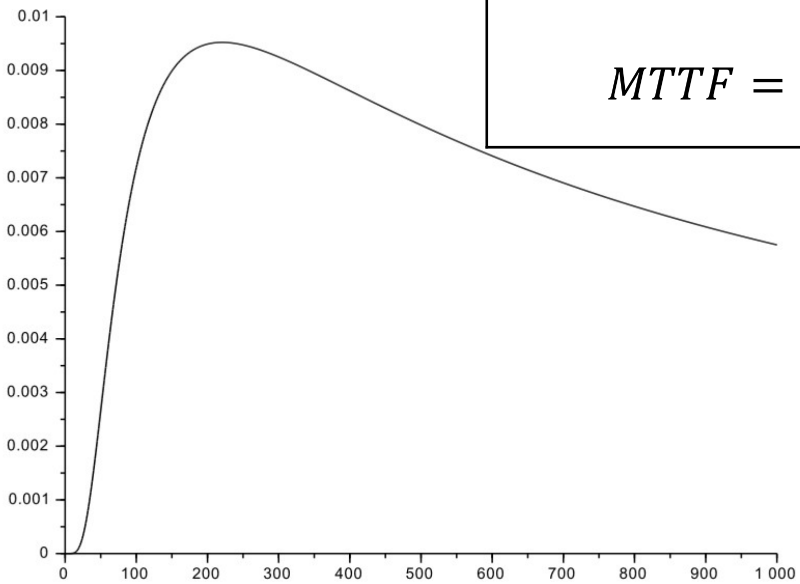
Definition

$$T \sim Lognormal(\mu, \sigma^2) \ if \ Y = \ln T \sim normal \ (\mu, \sigma^2)$$

$$f_T(t) = \frac{1}{\sqrt{2\pi}\sigma t} e^{-\frac{(\ln t - \ )^2}{2\sigma^2}} \ \text{if t} > 0$$

$$R(t) = \Phi\left(\frac{\mu - \ln t}{\sigma}\right) \qquad z(t) = \frac{\phi(\frac{\mu - \ln t}{\sigma})/\sigma t}{\Phi(\frac{\mu - \ln t}{\sigma})}$$

$$MTTF = e^{-\mu + \sigma^2/_2} \, ;$$



What does it fit for?

- Repair time
- Fatigue failure (e.g., in relation with the Whöler curve – strength and stress analysis)

Definition

A mixture model is when the sample comes from several distributions.

Examples:

- The analysis of multi-failure modes
- The mix of non-identical systems (sub-population)

Exercice:

Let consider a device with 2 independent failure modes. Each of the failure times are exponentially distributed with their respective failure rates λ_1 and λ_2. Define the system failure time and write the system reliability function